RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, AUGUST 2021
FIRST YEAR [BATCH 2020-23]

Date : 10/08/2021 **MATHEMATICS**
Time : 11am-1pm **Paper** : MACT 3 Full Marks : 50

Instructions to the students

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group A

Classical Algebra

Answer all the questions, maximum one can score 25.

1. Solve: $2x^4 + 6x^3 - 3x^2 + 2 = 0$. [7]
2. Solve: $x^3 + 9x^2 + 15x - 25 = 0$. [7]
3. If α be a special root of the equation $x^{12} = 1$, prove that $(\alpha + \alpha^{11})(\alpha^5 + \alpha^7) = -3$. [4]
4. Solve: $x^6 - x^5 + x^4 - 2x^3 + x^2 - x + 1 = 0$. [5]
5. If $\alpha, \beta, \gamma, \delta$ be the roots of $x^4 + px^3 + qx^2 + rx + s = 0$, find $\sum \frac{\alpha^2}{\beta}$. [3]
6. Show that the roots of the equation $x^4 - 12x^2 + 4 = 0$ are all real and distinct. [4]

Group B

Analysis 2

Answer all the questions, maximum one can score 25.

7. Prove or disprove the following statement:
   Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be uniformly continuous functions. Then $fg : \mathbb{R} \to \mathbb{R}$ defined as $fg(x) = f(x)g(x) \; \forall \; x \; \in \mathbb{R}$; will also be uniformly continuous. [4]

8. Let $f'(x)$ exists (and is finite) for $x \in \; (a-h, a+h)$ and f be continuous on $[a-h, a+h]$ where $h > 0$. Show that

   $$\frac{f(a+h)-2f(a)+f(a-h)}{h} = f'(a+\lambda h) - f'(a-\lambda h) \text{ where } \lambda \in \; (0,1).$$ [4]

9. If f has a finite third derivative f''' in $[2,3]$ and if $f(2) = f'(2) = f(3) = f'(3) = 0$; prove that $\exists \; c \; \in (2,3)$ such that $f'''(c) = 0$. [5]

10. In the following cases, give an example of a function f, continuous on S and such that $f(S) = T$, or else explain why there can be no such f:

    (a) $S = (0,1)$; $T = (0,1]$ [2]

    (b) $S = (1,4)$; $T = (0,5) \cup (7,10)$ [2]

11. Using the concept of open cover, prove that if K_1 and K_2 are compact sets in $\mathbb{R}$, then $K_1 \cup K_2$ is compact. [4]

12. Let I be an interval and let $f : I \to \mathbb{R}$ be differentiable on I. Show that if f' is positive on I, then f is strictly increasing on I. [4]

13. Using Cauchy condensation test show that $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges for $p > 1$. [5]

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